2601. Sketch the parametric curve, and find the t limits by setting $y = 0$. Set up a parametric area integral with
$$\int_{t_1}^{t_2} y \frac{dx}{dt}\, dt.$$
Evaluate with a calculator.

2602. Draw a clear sketch, including a radius to one of the corners of the rectangle. Use 1 and k as the half-sides and proceed by Pythagoras.

2603. Solve $x^2 - |x| = 0$, and exclude the solution set.

2604. (a) Define μ, with reference to the population, and state null and alternative hypothesis in terms in terms of μ.
(b) Find the critical value, and remember to state your conclusions (not too firmly) in context.

2605. Split the integrand into partial fractions:
$$\frac{2}{4-x^2} \equiv \frac{A}{2+x} + \frac{B}{2-x}.$$
Find A and B. Then integrate each fraction using the reverse chain rule and the standard result for the integral of $1/x$.

2606. (a) Translate the standard cubic $y = x^3$.
(b) Use the binomial expansion.

2607. In both (a) and (b), the primary reason is the need to restrict a many-to-one function to a one-to-one, and hence invertible, function. In (b), this is also combined with a linear transformation: consider the x domain if the range of $2x + 1$ is to be the domain of the arccos function.

2608. The error lies in the precise moment at which the second root is generated.

2609. Visualise the possibility space, either mentally or by drawing it for some value of n. Show that the number of successful outcomes is
$$1 + 2 + ... + n - 1.$$
Sum this as an AP and use $p = \frac{\text{successful}}{\text{total}}$.

2610. Assume the cube has side length 1. Then find the three lengths of triangle AMB, and use the fact that it is isosceles.

2611. This is a quadratic in $\sec x$.

2612. (a) Set $T = 0$.
(b) Find the acceleration of the car using the method of part (a). The boundary case is for zero speed at the top.

2613. Factorise this fully, and consider the multiplicity of the roots.

2614. Apply some of f, g, h or their inverses $\text{f}^{-1}, \text{g}^{-1}, \text{h}^{-1}$ to the equations. Start by rearranging ② and ③ for h(0). Then substitute into ①.

2615. (a) Use a standard log rule.
(b) Raise the base and input of a logarithm to the same power, and you don't change its value.
(c) Use the same technique as in (b).

2616. Differentiate and express dx in terms of u and du. Then substitute, including changing the limits from x to u. Multiply out and integrate.

2617. Substitute for t in the equation $x^2 + y^2 = 1$ of the unit circle. Solve this equation for t using a double-angle formula and the first Pythagorean identity. You're looking for a quadratic in $\sin^2 t$. Sub the values of t back in for the (x, y) coordinates.

2618. Argue directly from the conditional probability formula, using the fact that $\mathbb{P}(A \cap B) \neq 0$.

2619. Set up the gradient as $m = \frac{\Delta y}{\Delta x}$ and simplify using trig identities.

2620. Calculate the binomial probabilities.

2621. Factorise the quadratic expression on the LHS, and consider what must be true of the factors if their product is to be negative.

2622. A "proper" fraction is one which is not top-heavy. With polynomials, this means that the numerator has lower degree than the denominator.

2623. Use the first integral to find the y intercept of the line $y = \text{g}(x)$. Then substitute this, in the form $y = mx + c$, into the second integral to find the gradient m.

2624. Find the full probability distribution (probability table) of X, by drawing the possibility space of 24 equally likely outcomes. Then find $\mathbb{P}(00), \mathbb{P}(11)$, etc. by squaring. Add the probabilities.

2625. Enact the differential operators, and rearrange. Start with the operator inside the brackets.

2626. (a) Compare to freefall.
(b) Separate the variables to reach
$$\int \frac{1}{g - 0.1v}\, dv = \int 1\, dt.$$

Carry out both integrals. Then, to find the constant of integration, use the fact that the skydiver begins at (vertical) rest.

(c) Find terminal velocity by considering the limit $t \to \infty$; halve it and solve.

2627. Two are false and one is true.

2628. Consider the standard relationship between $\log_x 2$ and $\log_2 x$. To picture this, set $x = 8$.

2629. (a) This assumption allows modelling the tension as constant throughout the string.

(b) Assume that the stacked blocks move as one, and find the acceleration.

2630. (a) Consider that the roots of $ax^2 + bx - a^3 = 0$ must be $x = \pm 1$.

(b) Use the result from (a), and solve for roots. Choose between the resulting values of a by considering whether the parabola is positive or negative.

2631. Consider $r = \sqrt{x^2 + y^2}$ on an (x, y) plane. Then place this plane horizontally, and set $z = r$.

2632. Find each of the sets as a list of elements. The fact that the sets are written using x and y makes no difference to the lists.

2633. Consider an extreme example: $X_1 \sim B(1, 0.001)$ and $X_2 \sim B(1, 0.999)$.

2634. Consider the shaded region as a square plus four segments. Find the positions of the intersections of the arcs, and thus the angle subtended by each segment at the centre of its arc. Use these to find the areas of the four segments; add these to the area of a square.

2635. Complete the square to sketch the first curve. To sketch the second, transform $|x| + |y| = 4$. You can sketch this by considering four different line segments, one in each quadrant.

2636. Solve simultaneously, and look for a double root.

2637. (a) Find the number of ways of selecting the suits, and then of selecting the cards from those suits.

(b) Use the same technique as in (a).

(c) Divide your answers.

(d) Explain how you could have reached this result immediately via a short cut.

2638. For f to be polynomial, the denominator would have to be a factor of the numerator.

2639. (a) Differentiate and find m in terms of p; then find c in terms of p.

(b) Sub each point into the normal equation and solve for p. Then use Pythagoras.

2640. Assume, for a contradiction, that the third vertex lies at (p, q), where q is an integer.

2641. Use a standard result, and the reverse chain rule.

2642. (a) Consider the limit as $\varepsilon \to 0$.

(b) Use the quadratic formula.

(c) Consider the limit of the numerator.

(d) Consider the denominator.

2643. Compare to the chain rule.

2644. (a) Use a definite integral.

(b) Set your result from part (a) equal to 7390, and solve.

2645. (a) Solve by elimination.

(b) Differentiate implicitly.

(c) Use circle geometry.

(d) Use part (c).

2646. Such implications only hold if the second factor can never be zero. Think carefully about the statement "An exponential can never be zero."

2647. (a) Condition on bogus/genuine.

(b) Add the probabilities of two branches.

(c) Restrict the possibility space to the two branches above.

2648. Consider the largest circle, i.e. radius which can rotate inside an equilateral triangle when centred at its centre. Then convert to a square.

2649. Find the equations of the perpendicular bisectors of the easiest pairs, which are $(1, 1)$ and $(5, 1)$ and then $(1, 1)$ and $(4, 4)$. Solve simultaneously for the centre, and check that the remaining point is also equidistant.

— Alternative Method —

You could use the cyclic quadrilateral theorem, but it's considerably more effort here than the above technique.

2650. Take natural logs, and rearrange.

2651. In each case, consider the transformation(s) which take the signed area represented by b onto the signed areas required.

2652. Complete the square for x and y. An ellipse of the form $p^2(x-a)^2 + q^2(y-b)^2 = r^2$ has centre (a, b).

2653. Average a is the gradient of a chord; instantaneous a is the gradient of a tangent.

2654. Although the denominators look rather unfamiliar, this is standard partial fractions.

2655. This isn't true. Find a counterexample.

2656. The implication doesn't go both ways.

2657. This is a quadratic in $\tan x$.

2658. Resolve the initial velocity, and set up vertical and horizontal *suvat*. Rearrange the horizontal to make t the subject, then sub into the vertical.

2659. Consider the combined effect of the two (output) transformations: a reflection and a translation.

2660. Differentiate. Then, using $y - y_1 = m(x - x_1)$, set up the equation of a generic tangent. Solve two such equations simultaneously.

2661. (a) Use the standard formula.
   (b) Find out if $f''(1)$ is +ve or −ve. Use the fact that there are no points of inflection on $[1, 2]$ to prove that $f''(x)$ has the same sign throughout.

2662. Consider the degree of the equation $f(x) = g(x)$.

2663. The expected value is $\mu = np = 1.5$. So, check the probability of the outcomes 1 and 2.

2664. Use the fact that the roots are in AP to set out a symmetry argument. You don't need to calculate any integrals, although you could.

2665. Use $\mathbb{P}(\text{at least one}) = 1 - \mathbb{P}(\text{none})$.

2666. This is a disguised quadratic in 2^x.

2667. (a) Use the function facility of your calculator to check the predicted outputs.
   (b) Find the stationary point by differentiation.

2668. (a) The relevant fact is the number of roots.
   (b) Find the stationary point by calculus, and then use the factor theorem.

2669. Find the ranges of the denominators.

2670. (a) Multiply along the branches.
   (b) Restrict the possibility space to the first and third branches.

2671. (a) Take out a factor of x^2 inside the square root.
   (b) The generalised binomial expansion, which converges for $|x| < 1$, is

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + ...$$

   In this case, $\frac{p}{x^2}$ plays the role of x.
   (c) Cancel a factor of p before taking the limit.

2672. Rearrange the double-angle formula to make $\cos^2\theta$ the subject. Then let $\theta = 4x + \frac{\pi}{12}$. Then integrate by the reverse chain rule.

2673. Consider negative x.

2674. Using the fact that the curve passes through the origin, solve to find k. Then consider points at which the tangent is parallel to x or parallel to y.

2675. This is a quartic in $(e^{2x}-1)$. Either set $z = e^{2x}-1$, or factorise directly.

2676. One direction of the implication "if and only if" is true, the other is false.

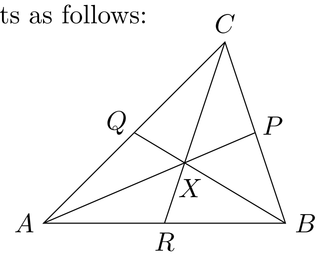
2677. Set the lengths as $1, r, r^2$, and use Pythagoras.

2678. Substitute into the LHS and RHS separately, and simplify both to give the same thing.

2679. (a) Quote a standard result.
   (b) Translation affects mean/variance differently.
   (c) Use the same result as in (a).

2680. (a) These are reflections of each other in $y = x$, so start by finding intersections with $y = x$.
   (b) Use the same fact as in (a).

2681. Label points as follows:



Show that $\triangle ARX$ and $\triangle BRX$ have the same area, and likewise for $\triangle AQX$ and $\triangle CQX$. Then show that $\triangle ARC$ has three times the area of $\triangle ARX$. Use this to show that $\triangle ARX$, $\triangle BRX$, $\triangle AQX$ and $\triangle CQX$ all have the same area.

2682. In fact, both students are right and wrong. Use a log rule to show exactly how the answers differ.

2683. There's no need to use the quotient rule here, and it's easier if you don't. Set $\mathrm{f}(x) = \sin^2 x + \sin x + 1$ and analyse the denominator on its own.

2684. Don't multiply the whole thing out. Instead, look for common factors first.

2685. An invertible function is one-to-one, which means that no line $y = k$ crosses the curve $y = \mathrm{f}(x)$ more than once.

2686. You don't need to do any calculations here.

2687. (a) Solve by substitution.
   (b) Consider the signs of each of the factors in the regions on your graph. You need exactly one negative.

2688. (a) Defining θ to be the acute angle between the F N force and the 200 N force, resolve parallel and perpendicular to the 200 N force. Take out a factor of $(F - 130)$ from each equation, and divide the equations.
   (b) Substitute back in.

2689. Since x gets simpler when differentiated, set $u = x$ and $\frac{dv}{dx} = \sin x$. Then use the parts formula.

2690. In general, three linear equations in two unknowns do not have a unique solution. However, that doesn't mean that such a thing is impossible.

2691. This is a common mistake: exponentiating terms rather than the sides of the equation.

2692. (a) Consider first the equivalent graph with local minima at $(3, 0)$ and $(5, 0)$.
   (b) Consider the symmetry of the quartic.

2693. Consider the vertex of the graph $y = \mathrm{f}(x)$.

2694. (a) Condition on having or not having the disease.
   (b) Restrict the possibility space to the positive branches.
   (c) Think about the implications if one has tested positive.

2695. Firstly, sketch an invertible version of $y = \sec x$, with domain and codomain restricted so that the graph is one-to-one. Then reflect in $y = x$.

2696. Find the magnitudes of the reaction forces between block and block, and block and table, and compare the values of F_{max}.

2697. Consider $\mathrm{f}(x) = x$ and $\mathrm{g}(x) = x^2$.

2698. The region is a square. Sketch the four boundary lines first.

2699. Start with the LHS side, and simplify with double-angle identities.

2700. Use the substitution $u = x + 1$. Show that the ratio of infinitesimals is $du = dx$. Rearrange to $x = u - 1$. Substitute all of these in to enact the substitution. Then multiply out the numerator and split the fraction up. At the end, re-substitute for u.

——— End of 27th Hundred ———